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A coherent state approach to effective potential in noncommutative $D = (2 + 1)$ models

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Abstract

In this work, we study the effective potential in noncommutative three-dimensional models where the noncommutativity is introduced through the coherent state approach. We discuss some important characteristics that seem to be typical to this approach, specially the behavior of the quantum corrections in the small noncommutativity limit.

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The noncommutativity of spacetime coordinates characterized by their commutation relation,

$$[\hat{q}_1^\mu, \hat{q}_2^\nu] = i\Theta^{\mu\nu}, \quad (1)$$

was originally proposed [1] as a way to avoid the ultraviolet (UV) divergences which arise within the perturbative approach to quantum field theory. Many years later, it was discovered that in a certain low energy limit of the string theory, a noncommutative field theory (NCFT) emerges [2]. This result inspired the scientific community to investigate different aspects of the NCFTs. One of the most intriguing issues in NCFT is the UV/IR (ultraviolet/infrared) mixing [3], that is the mixing of very distinct energy scales implying arising of the new so-called UV/IR infrared singularities, which can destroy the perturbative expansion. Another important issue is the lack of unitarity in theories where time does not commute with space coordinates [4]. This can be avoided via consideration of the theories with only space–space noncommutativity. Another observation is that relation (1) is not Lorentz invariant, unless we promote the noncommutative parameter $\Theta^{\mu\nu}$ to be an operator [5, 6], in contrast with the constant noncommutativity. One should observe that the noncommutativity of spacetime coordinates, in general, does not exclude the possibility for the appearance of UV divergences, and a renormalization prescription is still necessary to guarantee the consistence of NCFT.

Recently, one of the subjections to implement the noncommutativity of the spacetime coordinates, that is the coherent state formalism [7–9], has attracted some attention because

it apparently solves the problems with respect to unitarity, Lorentz invariance, and even the finiteness problem. The finiteness is achieved because the coherent state approach to NCFT introduces a natural cutoff Θ , that is the parameter of the noncommutativity, in the propagators of the model making any Feynman amplitude to be finite, at least while Θ is nonvanishing. In the three-dimensional case that we consider here, the coherent states naturally emerge in the context of anyons [10].

One remarkable result is the renormalization of the $D = (2 + 1)$ Gross–Neveu model, whereas the only consistent noncommutative extension of this model was obtained through a coherent state approach [11]. A large number of applications of this formalism have been studied, such as Aharonov–Bohm scattering [12] and some black hole effects [13–15].

In this work, using the tadpole method [16], we will study the first quantum correction to the effective potential in a noncommutative three-dimensional spacetime based on the coherent state formalism, discussing some issues and difficulties of the interpretation that was found in our approach.

In the coherent state formalism the commutation relation, equation (1), between the coordinates q_1 and q_2 implies that the variable $\hat{z} = \frac{1}{\sqrt{2}}(\hat{q}_1 + i\hat{q}_2)$ and its complex conjugate $\hat{z}^\dagger = \frac{1}{\sqrt{2}}(\hat{q}_1 - i\hat{q}_2)$ satisfy the following commutation relation:

$$[\hat{z}, \hat{z}^\dagger] = \Theta, \tag{2}$$

from which we can define a vacuum state $|0\rangle$ as the state satisfying the relation $\hat{z}|0\rangle = 0$ [7–9].

A coherent state is defined as an eigenstate of the annihilation operator \hat{z} given by

$$|\alpha\rangle = \exp\left(-\frac{|\alpha|^2}{2}\alpha\hat{z}^\dagger\right)|0\rangle, \tag{3}$$

where $\hat{z}|\alpha\rangle = \alpha|\alpha\rangle$.

If $\alpha = x + iy$, where x and y are commutative coordinates, a classical field $\phi(x)$ will be represented by

$$\phi(x) = \langle\alpha|\phi(\hat{q})|\alpha\rangle = \int \frac{d^3k}{(2\pi)^3} e^{-ikx - \frac{\Theta}{4}|\vec{k}|^2} \tilde{\phi}(k), \tag{4}$$

where $\tilde{\phi}(k)$ denotes the Fourier transform of $\phi(x)$.

Thus, with the coherent state representation of the classical field $\phi(x)$, we can determine its propagator which turns out to be

$$\begin{aligned} \Delta(x - y) &\equiv \langle 0|T\phi(x)\phi(y)|0\rangle \\ &= \int \frac{d^3k}{(2\pi)^3} e^{-ik(x-y)} \frac{e^{-\frac{\Theta}{2}|\vec{k}|^2}}{k^2 + m^2}, \end{aligned} \tag{5}$$

where the chosen signature is $(- + +)$.

From the above equation, it is easy to see that the free Lagrangian of the theory within the coherent state approach can be formally represented in a similar form to [11],

$$\mathcal{L} = \frac{1}{2}\phi(x)(\square - m^2) e^{-\frac{\Theta}{2}\nabla^2} \phi(x). \tag{6}$$

Now, let us develop the perturbative approach to the quantum field theories within the coherent state formalism. As a first example, let us consider a very simple model defined by the action

$$S = \int d^3x \left\{ \frac{1}{2}\phi \square e^{-\frac{\Theta}{2}\nabla^2} \phi - 8g^2\phi^6 \right\}, \tag{7}$$

which describes a massless real self-interacting scalar field.

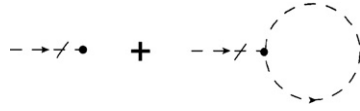


Figure 1. The one-loop tadpole equation. The dashed lines represent the scalar field ϕ propagator and the cut lines represent a removed external propagator.

To evaluate the effective potential, let us dislocate the field ϕ by Φ , $\phi \rightarrow (\phi + \Phi)$, where Φ is interpreted as the classical background field, and ϕ is a quantum one (cf [17]). So the action (7) can be rewritten in terms of new fields as

$$S = \int d^3x \left\{ \frac{1}{2} \phi (\square - M_\phi^2) e^{-\frac{\Theta}{2} \nabla^2} \phi - 8g^2 \phi^6 - 48g^2 \Phi \phi^5 - 120g^2 \Phi^2 \phi^4 - 160g^2 \Phi^3 \phi^3 - 48g^2 \Phi^5 \phi - 8g^2 \Phi^6 \right\}, \quad (8)$$

where $M_\phi^2 = 240g^2 \Phi^4$ is a background-dependent mass.

We will evaluate the effective potential by using the tadpole method [16]. The diagram that contributes to the tadpole equation at the one-loop order is depicted in figure 1, and the corresponding expression can be cast as

$$\begin{aligned} \Gamma_1^{(1)} &= -48ig^2 \Phi^5 + 480g^2 \Phi^3 \int \frac{d^3k}{(2\pi)^3} \frac{e^{-\frac{\Theta}{2} |\vec{k}|^2}}{k^2 + M_\phi^2} \\ &= -48ig^2 \Phi^5 - 120ig^2 \Phi^3 \frac{e^{\frac{\Theta}{2} M_\phi^2}}{\sqrt{2\pi\Theta}} \text{Erfc} \left[M_\phi \sqrt{\frac{\Theta}{2}} \right], \end{aligned} \quad (9)$$

where $\text{Erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-t^2} dt$ denotes the complementary error function.

Thus, the effective potential looks like

$$\begin{aligned} V_{\text{eff}} &= i \int d\Phi \Gamma_1^{(1)} = 8g^2 \Phi^6 + \frac{120g^2}{\sqrt{2\pi\Theta}} \int d\Phi \Phi^3 e^{\frac{\Theta}{2} M_\phi^2} \text{Erfc} \left[M_\phi \sqrt{\frac{\Theta}{2}} \right] \\ &= 8g^2 \Phi^6 + \frac{\sqrt{15}g}{\pi\Theta} \Phi^2 + \frac{e^{120g^2 \Phi^4 \Theta}}{4\Phi^2 \sqrt{2\pi\Theta^3}} \{\Phi^2 - \text{Erf}[\sqrt{120\Theta}g\Phi^2]\}, \end{aligned} \quad (10)$$

where $\text{Erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt = 1 - \text{Erfc}(z)$ is the error function.

Taking into account the asymptotic of the error function at the small argument,

$$\text{Erf}(x) \simeq \frac{2}{\sqrt{\pi}} \left(x - \frac{x^3}{3} + O(x^5) \right),$$

we find that the term Φ^2 completely disappears from the effective potential, which assumes its minimal value at $\Phi = 0$, so no spontaneous symmetry breaking is detected at one-loop order, just as it seems to happen in three-dimensional models which exhibit conformal invariance at the classical level [18–21].

As we suggest that the noncommutativity parameter Θ is very small, of the order of square of the Planck length, we can expand V_{eff} around $\Theta = 0$ and obtain

$$V_{\text{eff}} = \frac{1}{4\sqrt{2\pi\Theta^3}} + \frac{30g^2}{\sqrt{2\pi\Theta}} \Phi^4 + 8g^2 \left(1 - \frac{10\sqrt{15}}{\pi} g \right) \Phi^6 + 900 \sqrt{\frac{2\Theta}{\pi}} g^4 \Phi^8 + \mathcal{O}[\Theta]. \quad (11)$$

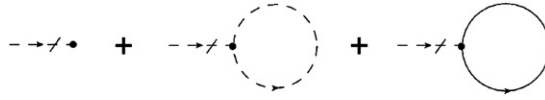


Figure 2. The tadpole equation for the Wess–Zumino model. The dashed lines represent the scalar field propagator and the solid lines represent the fermion field propagator.

We can see that the one-loop effective potential (11), obtained by using the one-loop tadpole equation (9), is finite while Θ differs from zero. The vacuum energy was shifted by $(4\sqrt{2\pi}\Theta^3)^{-1}$, but this is not an effective problem, because we can redefine V_{eff} to eliminate it. It is important to note that for very small Θ , $\Theta \ll g^2$, we find that the coupling $\frac{15g^2}{\sqrt{2\pi}\Theta}$ in the second term in equation (11) is much larger than 1, disabling a perturbative analysis of this effective potential; therefore, we cannot at least trust that our result is a good quantum approximation for the classical potential. This result can be naturally treated as an analog of the UV/IR mixing that haunts NCFT (cf [12] where the similar singularities arose within the coherent state approach to the Aharonov–Bohm effect). While this approach to NCFT keeps the Feynman amplitudes to be finite through a ‘natural’ cutoff, some remarkable aspects call the attention, which are, first, a very large vacuum energy $(4\sqrt{2\pi}\Theta^3)^{-1}$ and, second, a very large coupling constant arising due to the first quantum corrections.

It is well known that supersymmetry improves the UV behavior of ordinary theories, and noncommutative extensions of supersymmetric models are less dangerous in relation to UV/IR mixing than the extensions of non-supersymmetric ones. Therefore, as vacuum energy in supersymmetric models is known to vanish, see e.g. [22], let us test whether the bad issues of equation (11) arise in the three-dimensional Wess–Zumino model.

So let us consider the three-dimensional $\mathcal{N} = 1$ Wess–Zumino model [23] whose action being modified via the Gaussian factors within the coherent state approach is defined by

$$S = \int d^3x \left\{ \frac{1}{2}\phi(\square - m^2) e^{-\frac{\Theta}{2}\nabla^2} \phi + \frac{1}{2}\psi^a (i\gamma^\mu{}_a{}^b \partial_\mu + m\delta_a^b) e^{-\frac{\Theta}{2}\nabla^2} \psi_b - 8g^2\phi^6 + 6g\phi^2\psi^a\psi_a \right\}, \tag{12}$$

where spacetime indices are represented by Greek letters running from 0 to 2. Latin letters represent spinor indices assuming value 1 or 2.

To proceed with the calculation of the effective potential, let us shift the field ϕ by the background field Φ . Therefore, equation (12) can be written as

$$S = \int d^3x \left\{ \frac{1}{2}\phi(\square - M_\phi^2) e^{-\frac{\Theta}{2}\nabla^2} \phi + \frac{1}{2}\psi^a (i\gamma^\mu{}_a{}^b \partial_\mu + M_\psi\delta_b^a) e^{-\frac{\Theta}{2}\nabla^2} \psi_b - 8g^2\phi^6 - 48g^2\Phi\phi^5 - 120g^2\Phi^2\phi^4 - 160g^2\Phi^3\phi^3 + 6g\phi^2\psi^a\psi_a - 48g^2\Phi^5\phi + 12g\Phi\phi\psi^a\psi_a - 8g^2\Phi^6 \right\}, \tag{13}$$

where $M_\phi^2 = m^2 + 240g^2\Phi^4$ and $M_\psi = m + 12g\Phi^2$.

The diagrams which contribute to the tadpole equation for the Wess–Zumino model are depicted in figure 2. The bosonic contribution to the tadpole equation is given by

$$\Gamma_s^{(1)} = -i\Phi(m^2 + 48g^2\Phi^4) - 120ig^2\Phi^3 \frac{e^{\frac{\Theta}{2}M_\phi^2}}{\sqrt{2\pi}\Theta} \text{Erfc} \left[M_\phi\sqrt{\frac{\Theta}{2}} \right], \tag{14}$$

and the corresponding expression for the fermionic contribution can be cast as

$$\Gamma_f^{(1)} = 6ig\Phi M_\psi \frac{e^{\frac{g}{2}M_\psi^2}}{\sqrt{2\pi\Theta}} \text{Erfc} \left[M_\psi \sqrt{\frac{\Theta}{2}} \right]. \quad (15)$$

Integrating equations (14) and (15) over Φ and expanding around $\Theta = 0$, one finds the one-loop effective potential in the form

$$\begin{aligned} V_{\text{eff}} = & \frac{3g\Phi^2(4g\Phi^2 - m)}{\sqrt{2\pi\Theta}} + \left(\frac{m^2}{2} + \frac{3gm^2}{\pi} \right) \Phi^2 + \frac{m^2}{12\pi} (m - \sqrt{m^2 + 240g^2\Phi^4}) \\ & + \frac{4g^2}{\pi} \Phi^4 (9m - 5\sqrt{m^2 + 240g^2\Phi^4}) + 8g^2 \left(1 + \frac{18g}{\pi} \right) \Phi^6 \\ & - \frac{3g}{2} \sqrt{\frac{\Theta}{2\pi}} \Phi^2 (m - 4g\Phi^2) (m^2 + 12gm\Phi^2 + 192g^2\Phi^4) + \mathcal{O}[\Theta]. \end{aligned} \quad (16)$$

In the supersymmetric theories, the vacuum energy vanishes, i.e. for $\Phi = 0$ $V_{\text{eff}} = 0$. Nevertheless, it is important to note that the same problem with a large coupling $\frac{12g^2}{\sqrt{2\pi\Theta}}$ still arises. Moreover, if $m \neq 0$, the first quantum correction to the tree level mass is given by

$$\left. \frac{\partial^2 V_{\text{eff}}}{\partial \Phi^2} \right|_{\Phi=0} = m \left[-3g\sqrt{\frac{2}{\Theta\pi}} + m \left(1 + \frac{6g}{\pi} \right) + \mathcal{O}[\Theta]^{1/2} \right]. \quad (17)$$

Suggesting that $\Theta \sim M_P^{-2}$ (M_P is the Planck mass), one should note that the correction proportional to $\Theta^{-1/2}$ is negative; therefore, a tachyonic state arises when a tree level massive theory is considered unless $m > M_P$. But to deal with such massive particles, we should take into account the gravitational effects whose presence would radically modify our study; in particular, the Gaussian factor would also be modified. So it seems that the appearance of large effective coupling constants is a typical characteristic of the coherent state formulation of NCFTs.

We could rescale the coupling constant as $g = g'\sqrt{\Theta}$; in this case, we can be sure that a large coupling does not arise and no tachyonic state would emerge. But, do we expect that a very weak coupling after quantum corrections could generate a potential $g'\Phi^4$ with the coupling constant g' much larger than g ? Could this ‘miracle’ be a grasping of some unknown low-energy quantum gravity effect? A more profound study in other dimensions would be necessary to point a direction. Anyway, these simple models presented here show us some characteristics of the coherent state approach to NCFT which were not discussed before.

In this paper, we have calculated the one-loop effective potential in the noncommutative ϕ^6 theory and its supersymmetric extension formulated on the basis of the coherent state approach. The typical features of theories formulated within this approach are the singularities arising in the small Θ limit which can in principle imply in the tachyonic instability of the vacuum. Such singularities represent themselves as the natural analog of the infrared singularities arising within the UV/IR mixing mechanism in the usual noncommutative theories based on the Moyal product. However, we expect that better supersymmetric extension will allow us to rule out such divergences.

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